# Illuminated Choropleth Maps

A. James Stewart

School of Computing

Queen's University

Kingston, Ontario, K7L 3N6 Canada

(613) 533-3156 (Office)

(613) 533-6513 (Fax)

jstewart@cs.queensu.ca

Patrick J. Kennelly

Department of Earth and Environmental Sciences

CW Post Campus of Long Island University

720 Northern Blvd.

Brookville, NY 11050 USA

(516) 299-2652 (Office)

(516) 299-3945 (Fax)

Patrick.Kennelly@liu.edu

## Abstract

Choropleth maps are commonly used to show statistical variation among map enumeration units. Map makers take into account numerous considerations and make many decisions to produce a product that will effectively communicate spatially complex information to the map user. One design consideration is the choice between classed or unclassed choropleth maps. Unclassed maps assign a unique color, shade or pattern based on each unit's value. These maps are rich in information but may not be optimal for visual discrimination of regions or identifying values from a legend. Classed maps classify enumeration units based on unit values, and in some cases consider geographic area per class or contiguity. These classed maps better delineate regions and inter-class variation, but are designed to eliminate visibility of *intra*-class variations.

We present a method designed to use colors for choropleth classes, and soft shadows for intra-class variations. We conceptualize the choropleth data as a three dimensional prism model under simulated illumination, with the height of each enumeration unit a function of its mapped value. Our user studies have demonstrated that participants were able to use soft shadows to better identify which of two adjacent units was of greater population density, regardless of whether units were in the same or different classes. Additionally, the resulting soft shadows rarely interfere with the map reader's ability to match color classes to a legend, or to compare estimated differences in mean and variance of population density between two regions.

## Keywords

Choropleth maps, class intervals, illumination model, shading, shadowing.

# 1 Introduction

Choropleth maps show the variation in quantitative data among enumeration units such as countries, states or counties (Robinson et al., 1995; Slocum et al., 2008). The variation throughout the mapped area is displayed using such visual variables as hue, spacing, or lightness (Slocum et al., 2008). In this sense, such maps are geographic graphs, or spatially arranged displays of statistical data (Robinson et al., 1995).

The utility of a choropleth map to the map user depends upon a number of factors, both within and beyond the control of the cartographer. These include, but are not limited to, the geographic complexity of the phenomenon to be mapped, the decision to present data as either classed or unclassed, the method by which data will be symbolized, and the ability of the map user to interpret the resulting map. In our research, we focus on combining symbology from classed and unclassed data. We have devised a method to create illuminated choropleth maps in which soft shadows from an unclassed data model are used to add detail to classed choropleth maps symbolized by variations in hue. We have conducted user studies demonstrating that this technique significantly improves the map reader's ability to identify local variations between adjacent enumeration units. Additionally, our illuminated choropleth maps do not generally interfere with the map user's ability to match map colors to a legend, or to make regional comparisons of mean or variance between predefined geographic divisions.

## 1.1 *Literature Review*

The cartographic research of greatest relevance to our method can be categorized into two themes. The first focuses on the map user's ability to interpret map variations based on changes in choropleth symbols. This includes users outlining regions (e.g. Muller, 1979; Mak and Coulson, 1991), ranking map or region complexity (e.g. Olson, 1975), answering questions related to maps, regions, or particular units (e.g. MacEachren et al., 1998; Olson and Brewer, 1997; Brewer et al., 1997; Brewer and Pickle, 2002; Chang, 1978; Olson, 1975; ), matching unit symbology to legends (e.g. Brewer and Pickle, 2002; Mak and Coulson, 1991), and discriminating among units (e.g. Mak and Coulson, 1991).

The second theme, inextricably tied to the first, focuses on techniques designed to optimize the map maker's ability to display areas within regions as homogeneous and other areas as heterogeneous (e.g. Dent, 1999; Berry, 1968). This had led to an extensive discussion in the literature about how to best symbolize choropleth map units. Although statistical data were classified for map display since the first choropleth map in 1826 (Robinson, 1982), they only became a focus of study when Jenks and Caspell (1971) introduced methods to optimize the values of class breaks. Shortly thereafter, Tobler (1973) suggested assigning unique symbols to each enumeration unit to create unclassed choropleth maps.

### 1.1.1 Classed choropleth maps

Jenks and Caspell (1971) conceptualized the problem of finding optimal breaks as a series of three dimensional (3D) models. A classed choropleth map would have a 3D model which assigns the height of each class to the average value within the class. A second 3D model assigns height using each enumeration unit's value. Given this construct, the difference between the class mean and all enumeration unit values in the class can be measured and summed in various ways.

Jenks and Caspell (1971) summarized variations from the class mean using three different "error" metrics: tabular error, overview error and boundary error. Minimizing tabular error is the method described in current literature as the "optimal" method (Robinson et al., 1995, Slocum et al., 2008). It is based on the statistical research of Fisher (1958). Jenks and Caspell (1971) were the first to apply this research to defining class boundaries, an important basis upon which much future research was built. Jenks (1977) details the method of iteratively finding class breaks so that within-class variations from the mean are minimized. It looks only at the values and statistics associated with the tabular data; no geographic data are used.

Jenks and Caspell's (1971) overview error takes into account geographic area by calculating a "volume" based on the 3D models. Each enumeration unit has an area, and a "height" based on the difference between the class mean and its value. Variations of this volume can be minimized within each class in a manner similar to that described for tabular error. Although straightforward to implement using a geographic information system (GIS), most commercial algorithms do not account for overview error (Armstrong et al., 2003).

Other choropleth research focuses on issues related to overview error, not only on unit area but also on unit location. Monmonier (1972) was the first to tackle the issue of spatial clustering in creating clear and simple choropleth maps. Monmonier's concerns were justified later that decade by Chang (1978) who documented the preference of map readers for simpler, less fragmented map patterns. Monmonier (1972) used a taxonomic clustering algorithm to explicitly consider contiguity in selecting class intervals. He also included algorithms to balance statistical and geographical considerations. Olson (1975) used measures of spatial autocorrelation to analyze aspects of the overall look of classed maps. She found through user testing that concepts such as spatial complexity could be related to quantitative measures of autocorrelation.

Jenks and Caspell's (1971) third measure of error is boundary error. Returning to their 3D construct, ideal class boundaries would correspond to the enumeration unit borders with the largest changes in the mapped value. In other words, larger steps (or "cliffs") in the 3D enumeration unit model would ideally correspond to class boundaries. They state the following: "The boundaries between shading on a choropleth maps tend to dominate the visual impact of the representation, because sharp visual contrast occur along these lines. Map readers tend to assign significance to these boundaries and, as a result, often assume that they designate breaks in the configuration of the statistical surface." (p. 229). Their strategy was to summarize and maximize variations in values at class breaks using various classifications. Varying class boundaries to address boundary error resulted in variations in their other two measures of error. They chose to give equal weights to all three error measures to define an optimal solution.

Their discussion of boundary error is important to our research for two reasons. First, Jenks and Caspell (1971) minimized boundary error to address a specific concern: the potential misinterpretation that large variations always occur at class boundaries and lesser or no variations always occur within classes. Second, the issue of minimizing boundary error is related to spatial contiguity, but is not guaranteed to result in simpler, less fragmented map patterns as are the clustering methods discussed above. For example, a region might consist entirely of a single class as defined by tabular and overview error. If one enumeration unit is surrounded by units with lower value in that class, the central unit could be promoted to a new class upon addition of boundary error into the classification process. Such results could yield a more complex, fragmented map pattern.

Recent research identifies additional criteria that can be used to define class breaks. Cromley (1996) uses boundary error in a comparison of a number of classification methods. Armstrong et al. (2003) use a genetic algorithm which finds an optimal solution based on a number of criteria. They minimize measures of Jenks and Caspell’s tabular error, aerial inequality among classes, and boundary error as defined by MacEachren’s (1982) face complexity measure, as well as maximizing a reformulated Moran’s I as a measure of spatial autocorrelation. They do not assign weights to these factors, but rather find “Pareto optimal” solutions, ensuring that one criterion does not dominate another. The applications of evolutionary algorithms for comparing choropleth maps are discussed by Xiao and Armstrong (2005).

### 1.1.2 Unclassed Choropleth Maps

All class boundary research can be summarized as methods focused on finding the optimal (or set of optimal) class breaks based on one or more criteria. This research stems from the assumption that the values of a small number of class breaks is of vital importance. Other geographers suggest the possibility of diluting the importance of any particular class break by increasing their number, the logical limit being a different class for each value. This sort of map is referred to as an unclassed choropleth map. Tobler (1973) was the first to devise a method for creating such maps with a line plotter. Beginning with that article and comments of concern from Dobson (1973), the relative merits of classed versus unclassed choropleth maps have been the topic of much discussion.

Mueller (1979) tested the ability of users to categorize areas of high, medium and low density from unclassed choropleth maps of 1970 rural population density using the counties of Kentucky as enumeration units. Users closely replicated a choropleth map of the same data with three optimized classes. Muller (1979) argued that these results implied map users are able to identify regions from unclassed maps, and that such maps offer the additional benefit of reproducing the data on which the map is based. Dobson (1980) argued Mueller’s (1979) study focused on pattern delineation, without evaluation of more advanced map skills such as pattern memorization. Mueller (1980) responded by underscoring the importance of recognizing map patterns, and the suspect nature of class boundaries.

Results from more recent research underscore the complexity of issues involved in such comparisons. Gilmartin and Shelton (1989) found classed choropleth maps reduced map-processing time when compared to unclassed maps. Slocum et al. (2008) state that classed maps are generally more effective than unclassed maps for the acquisition of specific information. They make the point that “The high accuracy of unclassed maps is, however, mathematical, not perceptual.” (p. 267). Nonetheless, this conclusion is based on maps with few classes (classed) versus maps with many classes (unclassed). Even then, results of such studies as Mersey (1990) and MacEachren (1982) are inconclusive about the effect of variation in the number of classes on some cognitive skills, such as the recall of specific information. In a non-user based study, Cromley (2005) found more visual complexity of spatial patterns in classed versus unclassed maps by performing algebraic-to-graphic transformation lines to highlight the role of the maximum contrast principle.

### 1.1.3 Prism Maps

Our research focuses on adding more detailed information to classed choropleth maps using soft shadows from unclassed values. In these illuminated choropleth maps (**Figure 1**), the hues of the unit are based on the class. The shadows are based on an illumination model applied to a volumetric model of the enumeration units. In the volumetric model, each unit is extruded to a height based on the attribute being mapped. For a given illumination direction defined for the former model, the length of the shadows will be a function of the difference in values between adjacent units. These shadows act as a second unclassed visual variable, used with the intent of adding detail to classed choropleth maps in a manner that is perceptually intuitive.

Shadows are not defined as a visual variable, although they are often represented by changes in lightness. In the manner in which they vary, they are most similar to "perspective height," identified as a visual variable by Slocum et al. (2008). The perspective height variable extrudes area symbols into the third dimension based on value of some attribute. Such a "prism" display was used by Jenks and Caspall (1971) in Figure 1of their seminal article (**Figure 2a**). In this example, shadows are not used to enhance the 3D effect; black areas represent sides of units. Also, Jenks and Caspell (1971) made no attempt to combine these extruded maps with classed choropleth maps such as their Figure 2 (in our **Figure 2b)** in a single display.

Such prism maps have the advantage of an excellent 3D effect. Interest in constructing such maps led to development of optical (Jenks and Brown, 1966) and computer automated (e.g. Franklin and Lewis, 1978; Hilbert, 1981) techniques. Prism maps continue to be popular today, especially in the mass media. Prism maps, however, also have disadvantages. Any map that is not planimetrically correct will have increased distortion in shape, size, distance and direction. Additionally, some units may be hidden from view, and these are likely to change based on viewing direction. All of these factors may make typical choropleth map uses (such as identifying local or regional variations) more difficult.

Some of these issues were addressed by a method using stereoscopic vision to create 2D choropleth maps with a true 3D appearance (Jensen, 1978). Such maps overcome many of the issues of traditional prism maps. Users of such maps, however, will still be faced with the need for stereoscopic vision and the challenge of a limited field of view with such maps.

Our illuminated choropleth maps are planimetrically correct. Since hard shadows would obscure some units, we focus on creating *soft shadows* that vary the tone of class colors in a subtle manner. We use rigorously defined illumination models to match theoretical results. In doing this, we are attempting to create a map display that is easily and intuitively visually interpreted. We realize that other methods, such as labeling population density values for each polygon, could provide even more information and can be an effective practice for maps with somewhat limited numbers of polygons, such as the states of the United States. We do not, however, feel that this would be a visually effective way to display maps with much more numerous polygons, such as our examples using counties of the United States. Finally, we conduct testing to ensure that users can correctly interpret shadows with respect to local variations, and that shadowing does not interfere with users' ability to match unit colors to a legend or to make regional comparisons of mean and variance among large areas.

Our illuminated choropleth map, in its use of an attribute being displayed using multiple visual variables, shares similarities and has important differences when compared with traditional cartographic techniques. It is similar to a bivariate choropleth map, but our method maps only one attribute in two manners. It is also similar to maps of smoothly varying statistical surfaces such as topography that employ layer tinting with hill shading and shadowing.

### 1.1.4 The Bivariate Map Analogy

Bivariate choropleth maps combine the display of two attributes on the same map. Initial bivariate maps focused on creating a matrix of easily identified color classes. Olson (1981) concluded that, although not without issues, students did gain information from such maps and find them interesting and appealing.  Eyton (1984) devised a method in which the center of the legend-matrix is gray, with corners on the diagonals comprising two complementary colors, and black, and white. Brewer (1994) devised color schemes that take into account unipolar or bipolar attributes.

Other studies used an unclassed approach to bivariate choropleth maps. Techniques focused on the use of cross-hatched lines, with the spacing of horizontal and vertical lines varying with the two attributes of interest (Carstensen; 1986a; Carstensen; 1986b; Lavin and Archer, 1984;

Carstensen, 1982). The resulting maps display tonal variations as well as variations in the size, dimension and orientation of individual rectangles.

Illuminated choropleth maps display one attribute using two different visual variables. Although colors are assigned based solely on, in our example, population density values, shadows will change depending on conditions in the neighborhood. For example, an enumeration unit may cast a very short shadow if its neighbor in the direction opposite the illumination direction is nearly as densely populated. A unit in the most densely populated class might cast a long shadow if its neighbors have significantly lower population densities. Long shadows are not necessarily limited to the adjacent neighbor, depending again on local conditions.

Our tests indicate users are not generally bothered by these shadows in matching class colors to a legend or making regional evaluations of mean and variance. Results are consistent with other perceptual studies which indicate that users are able to see continuous patterns through shaded regions, even if the patterns are represented as shades of gray (Adelson, 2000; Aldelson, 1993). We suggest that shadows on illuminated choropleth maps do not offer a perceptual challenge to users because they are based on a 3D model illuminated in a predictable manner.

### 1.1.5 The Topographic Map Analogy

Early researchers pointed out the similarity of choropleth and topographic maps. Jenks and Caspell (1971, p. 218) stressed the impression a choropleth map will have on the map reader: “First, he may seek an overview of the statistical distribution from the choropleth map, much as he obtains the ‘lay of the land’ from a topographic map.” Monmonier (1972) endorsed symbology that helps to display choropleth maps simply and clearly. He drew analogies to the varying contour intervals and classes of hypsometric tints used to create topographic maps that appear spatially organized. Tobler (1973) drew an analogy between selecting larger class intervals to generalizing a topographic surface by, for example, choosing a large contour interval.

We use contour mapping to discuss some of the similarities and differences between topographic and choropleth maps. Contour maps are often displayed with elevation values assigned to hypsometric or layer tints in a manner similar to which colors are assigned to classed choropleth maps. Contour maps represent the surface by a series of lines representing intersections of the terrain with planes evenly spaced in elevation; choropleth maps represent statistical variations by enumeration units which may vary in any manner. The former is appropriate for representing a surface of smooth variation, the latter for a surface of irregularly steps between otherwise flat surfaces (i.e. the tops of the prisms). Given such a construct, a prism map would have contours on its vertical faces (e.g. Franklin and Lewis, 1978).

Cartographic techniques include other methods for mapping terrain, such as hill shading and associated shadowing. Hill shading generally uses a simple directional illumination model to vary the shade of gray of individual map units (Imhof, 1982; Horn, 1982). The shade of gray is determined by the angular difference between the direction of illumination and the surface normal. Hill shading was first automated by calculating shading values for a small grid (Yoeli, 1965), relating these shades to the density of black dots on a white background (Yoeli, 1966), and using a computer-controlled electronic typewriter to print and overprint characters to match desired hill shades (Yoeli, 1967). Subsequent efforts led to use of special characters on a line printer (Brassel et al., 1974) and finally continuous shades of gray on gray tone plotters (Peucker et al., 1974). This type of hill shading would be ineffectual for choropleth maps, as all enumeration units have the same (horizontal) orientation.

The same directional illumination model can also be used in terrain mapping to define areas in shadow. Although shadows provide important visual cues to local relief, they have a poor reputation in cartography because of their tendency to obscure local details (Imhof, 1982). Applying a simple directional illumination model to the data in this study, we get a map with dark, sharp, hard shadows obscuring more areas (**Figure 3**). More sophisticated clear sky illumination models allow units beneath soft shadows to cast their own shadows, as diffuse illumination from other sectors of the sky is not obscured. This is evident comparing the clear sky and directional shadowing in **Figure 3,** as no units in cast directional shadows are creating their own shadows.

Additionally, the illumination model can serve to shade flat units. This shading results from diffuse light distributed throughout the sky being partially blocked at certain locales by high areas in the prism model that are not in the line of sun illumination. An example of an urban elevation model with equal illumination from all directions shows such shading patterns (Kennelly and Stewart, 2006). Flat tops of buildings are rendered in many shades of gray, depending on how much of the virtual sky is obscured by other buildings. The implication for illuminated choropleth maps is that units more obscured will be shaded slightly darker than surrounding units in the same class, even if they are not covered by an obvious shadow. An example of this effect is the relatively darker shade of some class colors in northeastern counties, which are not apparently beneath soft shadow.

In summary, illuminated choropleth maps and topographic maps with layer tinting and hill shading represent very different types of geographic phenomena, but similar shading methods can be used to represent them. In terrain maps, elevation layer tinting applies the same colors over continuous regions within specified ranges of values. These colors are modulated by a derivative map based on a directional illumination model that defines shading based on local relief. Our illuminated choropleth method applies the same colors over potentially less continuous, stepped surfaces within specified classes. These colors are modulated by a different derivative map based on a more sophisticated illumination model that defines soft shadowing based on local attribute variations.

# 2 User Study

## *2.1 Methods*

We began with U.S. Census Bureau county data for the conterminous United States in a geographic information system-based vector format. Data were taken from the last decennial report of 2000, and were projected into an Albers equal area map projection. As well as counties, independent cities are included as county equivalents. We calculated population density in people per square mile for the 3,184 polygons representing 3,109 counties or county equivalents, and converted data into a one kilometer resolution raster format. Using this sample size, all polygons are represented by at least one grid cell. We used this grid as input for a custom application written in the C++ programming language to “illuminate” the choropleth map, as described in Section 2.2.

Population density values ranged from 0 to 55,092 individuals per square mile. We classified the data into five categories. One of the goals of our study was to evaluate the user’s ability to discern randomly selected units of various class colors with soft shadows. The Jenks optimal method, however, did not lend itself to this sort of analysis. Using the Jenks method, for example, only four clustered county equivalents of relatively small size fall into the class of highest population density of more than 21,170 (the New York City boroughs of Manhattan, Brooklyn, the Bronx and Queens) (**Figure 4**).

We chose to define our own class breaks so that a greater number of counties or county equivalents of different classes would be distributed across the map. Although similar to concerns of overview error discussed previously, we did not optimize reduction of this error, as our goal was not to balance volumes within each class, but to make available more locales of classes that might be randomly sampled. Beginning with initial Jenks optimal breaks at 805, 3166, 8379 and 20705, we adjusted class breaks to 15, 60, 200, and 1500 (**Figure 5**).

We applied a sequential color scheme based primarily on changes in hue from yellow through orange to red to these classes (Brewer, 2005, Brewer et al., 2003, Harrower and Brewer, 2003). We opted to keep the lightness of all colors relatively high, with the Value of the Hue-

Saturation-Value specifications remaining above 97%. This would allow nearly the entire gamut of Value variations as soft shadows to overprint the colors in the final color version of the map. It can be noted, however, that colors ranging around the color circle from yellow to red have very different luminosities (Slocum et al., 2008; Kennelly and Kimerling, 2004; Brewer, 1994). While changing Value of our light yellow from 0% to 100% varies luminosity by about 98%, doing the same to red only varies its luminosity by about 55% , as red is a less luminous color. Assigning red to our areas of highest population density minimizes issues associated with its decreased dynamic range of luminosity, as these units are more likely to cast shadows than be partially obscured by them.

### *2.2 Illumination Model*

Many realistic sky models have been developed. The classic "overcast sky" of Moon and Spencer (1942) provides the sky radiance in a particular direction as

$$0.33\, L_Z \left( 1 + 2 \sin\theta \right) \qquad \text{Equation 1}$$

where $L_Z$ is the radiance at the zenith and $\theta$ is the angle between the particular direction and the horizontal. This model provides three times as much illumination at the zenith as at the horizon. The CIE Standard General Sky (Commission Internationale de l'Eclairage, 2001) provides a much more elaborate formula in which five parameters may be set to model various skies, from clear to partly cloudy to overcast (Darula and Kittler, 2002). The model we used for **Figure 6** is a "clear day" illumination model.

In our illuminated choropleth maps, the gridded data were treated as a surface, with the surface height at each point being a function of the population density at that point. The surface was illuminated with a clear day sky in which most light arrived from all directions equally, with 21 times as much light arriving from the direction of the sun, which was placed at a 45 degree elevation above the horizon in the northwest. The radiance of the sky in a particular direction was

$$0.05 + \cos^{500} \theta \qquad \text{Equation 2}$$

where θ is the angle between the particular sky direction and the sun. The cosine term, with its high exponent, ensured that the brightest light came from the direction of the sun, and that the sun's contribution tapered to zero at about seven degrees away from the sun's direction. This resulted in soft shadows at the base of "cliffs" on the surface (caused by the decrease in the amount of visible sky and hence, total shadowing, at the cliff base), and a somewhat diffuse shadow cast from the sun by high parts of the surface.

The computation of the surface illumination was done using the method of Kennelly and Stewart (2006). Their method computes an approximation of the horizon at each grid point. Using the horizon, their method computes the illumination at a grid point by integrating the sky illumination over the sky area that is above the horizon, and reflecting that light in proportion to the surface albedo (0.6 in our case). We scaled the computed surface illumination to the range 0 to 255 so that it could be represented compactly as a gridded map display.

Due to the discrete nature of the horizon approximation, long shadows can appear somewhat fanned out, especially in the vicinity of localized spikes in the surface. That effect can be reduced with greater computation time in the horizon algorithm (to produce a more accurate horizon), but the subjects in our study did not report this to be distracting.

Our first attempts at illuminating the surface produced poor results. Most of the height differences between adjacent counties occurred in lower, less dense parts of the surface (particularly in the Midwestern states) and the scale of those differences was insignificant compared to the scale of the largest heights. Those very small differences were insufficient to cast shadows.

In order to enhance small height differences and, hence, to cast shadows in these areas, we transformed the heights with an exponential function that increased small heights more than it did high heights, and that scaled the heights so that they were of the same scale as our map. We used the formula

$$H = 0.0025\ (D / D_{max})^{0.455}\ D_{max} \qquad \text{Equation 3}$$

where H is the new "height" of the unit being scaled (measured in people per square mile), $D_{max}$ is the maximum population density of all units, and D is the original population density of the particular unit being scaled. This transformation increases the subtle differences between the many units of low population density. At the same time, it suppresses the units of highest population density, those most likely to cast long shadows prone to artifacts of the method.

These new heights were used only for the illumination model and the resulting shading. Classes were assigned based on the original, unadjusted values of population density. The height transformation function is shown below as **Figure 7**.

### *2.3 User Testing*

The illuminated choropleth method provides a fine-grained view of the data (**Figure 6**). Combining these soft shadows with classes symbolized by color reveals relative values within the same class, especially between adjacent polygons (**Figure 1**). The illuminated method also lets us see the underlying enumeration units within a class when those samples are sufficiently different to cause shadowing. These assumed benefits are balanced by potential disadvantages. First, the shadowing may obscure the class coloring. Second, the shadowing may confound the perception of aggregated characteristics, such as the mean or variance of a region.

We performed a formal user study to determine whether these advantages and potential disadvantages were statistically significant. We made the following hypotheses:

H1: The illuminated method improves a person's ability to determine which of two adjacent counties has a higher population density.

H2: The illuminated method does not affect a person's ability to determine the class of a county, given a map legend.

H3: The illuminated method does not affect a person's ability to determine which of two aggregate regions has a higher mean population density or higher variance in population density.

## 2.4 Test Cases

A computer program was written in C++ to draw on a computer screen a choropleth map of population densities in the conterminous United States, both with and without illumination. The program randomly selected sites (counties, pairs of adjacent counties, or US Census Bureau divisions) zoomed to the area of interest, highlighted the sites of interest, recorded mouse clicks, and kept track of response time. The program operated in four modes, corresponding to four different tasks. For each task, a part of the map was shown shaded or unshaded and the subject was asked to click. The four tasks were:

### 2.4.1 Pair Selection Task

Two adjacent counties were centered on the screen and highlighted with blue circles. The subject was asked to click on the county of higher population density. In some cases, the selected, adjacent units were in different classes (**Figure 8**). In other cases, the units were in the same class (**Figure 9**).

### 2.4.2 Legend Matching Task

A county was centered on the screen and highlighted with a blue circle. A legend of class colors was shown on the upper-right corner of the screen. The subject was asked to click in the menu on the color that matched that of the highlighted county (**Figure 10**).

### 2.4.3 Region Mean Task

The conterminous United States was shown divided into the eleven U.S. Census Bureau divisions and spatially separated for visual clarity. Two randomly selected regions were highlighted, with all other regions assigned one shade of gray. The subject was asked to click on the region of greater mean population density (**Figure 11**).

### 2.4.4 Region Variance Task

The same method was used to display two divisions. The subject was asked to click on the region of greater variance. The presentation was identical to **Figure 11**.

## *2.5 Experimental Setup*

We tested 41 subjects who were faculty, graduate students, and staff in geography and computer science department. Subjects were divided into **experienced** and **non-experienced** groups: We considered as experienced those subjects with some graduate experience in computer science graphic rendering with shading and those subjects with some graduate experience in geography with cartographic hill shading. Of the 41 subjects, 16 were experienced and 24 were non-experienced.

Each subject performed four groups of tasks, in this order:

1. Legend matching tasks for 18 counties. Each county was shown once shaded and once unshaded for a total of 36 tasks.
2. Pair selection tasks for 12 pairs of adjacent counties from the same class, and for 12 pairs of adjacent counties from different classes. Each pair was shown once shaded and once unshaded, for a total of 48 tasks.

3. Region mean tasks for 15 pairs of regions, with each pair shown once shaded and once unshaded.
4. Region variance tasks for 15 pairs of regions, with each pair shown once shaded and once unshaded.

Within each of the four groups of tasks above, the same tasks (i.e. the same counties, county pairs, or region pairs) were used, but the order of the tasks was randomized to reduce learning bias.

Before a subject's trial, the class colors were explained to the subject, as was the "higher is denser" representation used in the illumination method. Each subject was trained on four matching tasks, six pair selection tasks, four region mean tasks, and four region variance tasks. Variance was explained as being the degree to which densities varied from the mean. During the trial, the subject's responses were recorded for later analysis. Each trial took approximately twenty minutes. After the trial, the subject was asked for subjective evaluations of the illumination.

## *2.6 User Study Results*

The two conditions under which each task was performed were "with illumination" and "without illumination." For each task, a subject's **performance** was defined as the fraction of correct responses made by the subject. A subject's **completion time** was defined as the time between the presentation of the task on the screen and the subject's subsequent mouse click.

Student's t-test for paired data was used to determine whether the subjects performed differently under the two conditions. Two one-sided tests (TOST) were made to determine whether subjects performed equivalently. We considered a difference in performance of up to ten percent to be equivalent. In the following results, we show the **95% confidence interval** of the mean of each measure as **mean ± 1.96 standard error**. All results are summarized in Table 1.

### 2.6.1 Pair Selection Task

Subjects performed significantly better ($p < 0.02$) at selecting the denser of two adjacent counties *of the same class* when using illumination (accuracy 0.75 ± 0.09) than when not using illumination (accuracy 0.65 ± 0.04). The performance increase was even more substantial among experienced subjects (0.88 versus 0.67, $p < 0.001$). No conclusion could be drawn about the relative performance of non-experienced subjects under the two conditions.

We also include histograms to highlight the differences we detected with the summary statistics above (**Figure 12**). Without shading, subjects have a fairly equal distribution of correct choices, with scores varying between 40% and 90%. With shading, over half (21) of the subjects scored in the 90% - 100% range (with 13 perfect scores). Another point to note is that no subjects scored terribly low without shading. With shading, two subjects scored less than 10%. These subjects were not experienced with shading techniques, and may have mentally inverted the 3D model.

Also worth noting is that subjects scored much better (accuracy 0.65) than random chance (accuracy 0.50) when not using illumination, even though there was no difference in the appearance of the two counties. Two subjects reported that they used the strategy of picking, as denser, the county that was closer to a major population center.

For adjacent counties of the same class, subjects took more time when using illumination (3.0 ± 0.7 seconds) than when not using illumination (2.5 ± 0.5 seconds). The difference was significant ($p = 0.010$).

Subjects performed about the same at selecting the denser of two adjacent counties of *different classes* when using illumination (accuracy 0.92 ± 0.03) than when no using illumination (accuracy 0.94 ± 0.02). The two conditions were equivalent ($p < 0.001$) for an effect size of ten percent, regardless whether the subject was experienced or non-experienced.

For adjacent counties of different classes, subjects took more time when using illumination (2.2 ± 0.3 seconds) than when not using illumination (1.9 ±0.2 seconds). The difference was significant ($p < 0.001$).

### 2.6.2 Legend Matching Task

Subjects performed equivalently at classifying counties when using illumination (accuracy 0.89 ± 0.04) to when not using illumination (accuracy 0.91 ± 0.03). The two conditions were statistically equivalent for both experienced ($p < 0.002$) and non-experienced ($p < 0.003$) subjects. Subjects took equivalent time to classify when using illumination (3.41 ± 0.42 seconds) and when not using illumination (3.38 ± 0.35 seconds) ($p < 0.029$) considering an effect size of ten percent (i.e. 0.34 seconds).

We did, however, notice that subjects performed poorly when classifying a few particular counties: those that fell in the deep shadow of a much denser adjacent county. With the sun placed in the northwest, deep shadows were cast on counties to the southeast of counties with much greater population densities. An example of one such site is shown in **Figure 13**. In these cases, the subjects had a mean accuracy of 0.40 $\pm$ 0.15 with illumination and 0.95 $\pm$ 0.07 without illumination. The difference was significant ($p < 0.001$).

### 2.6.3 Region Mean and Variance Tasks

The two region tasks required the subjects to roughly estimate the aggregate measures of mean and variance for regions consisting of many counties. Given a pair of regions, subjects were asked to click on the region of greater mean or variance.

For the region *mean* task, there was no significant difference in performance with illumination (accuracy 0.97 $\pm$ 0.01) or without illumination (accuracy 0.97 $\pm$ 0.02). The two conditions were statistically equivalent ($p < 0.001$). For the region mean, subjects took more time when using illumination (3.7 $\pm$ 0.6 seconds) than when not using illumination (3.3 $\pm$ 0.4 seconds), although no statistical conclusion could be reached.

For the region *variance* task, subjects had much more difficulty, although again the performance was equivalent ($p < 0.001$) with illumination (accuracy 0.61 $\pm$ 0.06) and without illumination (accuracy 0.63 $\pm$ 0.06). For the region variance, subjects took more time when using illumination (5.1 $\pm$ 0.6 seconds) than when not using illumination (4.5 $\pm$ 0.7 seconds). The difference was significant ($p = 0.042$).

All of the subjects reported that they found the region variance task to be much more difficult than the other tasks. Five subjects reported that they tried to envision histograms of the class frequency for each region and to estimate the variance from the histogram ... a daunting mental task.

### 2.6.4 Post-trial Survey Responses

After the trial, each subject was asked to evaluate three statements on a five-point Likert Scale with responses “strongly agree” (SA), “agree” (A), “neutral” (N), “disagree” (D), and “strongly disagree” (SD). Responses from all 40 subjects were gathered. The statements are presented below, and user responses are presented in **Figure 14** for statements (A), (B), and (C) listed below.

(A) “The illuminated choropleth map gave me a better understanding of variations within a region (consisting of many counties) than did the unshaded choropleth map.”

(B) “It was more difficult to determine the class of a county in the illuminated choropleth map than in the unshaded map.”

(C) “It was easier to see the boundaries of counties on the unshaded choropleth map, compared to the illuminated choropleth map.”

Subjects were also asked if they had any further comments about the illuminated choropleth map. Several subjects said that the shaded map “was visually nice” or “looked more accurate” or “was easier to interpret”. One comment came in various phrasings from four subjects, who said that the color cue, where deeper colors represent regions of denser population, competed with the illumination cue, where lighter shades correspond to higher, denser regions that are not in shadow. One subject said that the orange class colors were “close” and hard to distinguish in shadow.

# 3 Discussion

The real strength revealed by this study is the users’ ability to use soft shadows to identify local variations within classes. It is readily apparent that an illuminated choropleth map is more detailed than its counterpart without shadows. Our study shows that this detail adds information to the map in a manner that many users, especially experienced subjects, are able to understand. We see the statistically significant improvement in performance of non-experienced subjects as an indication of the intuitive nature of shadows, and the even greater improvement in performance of experienced participants as an indication of a capacity to learn to interpret such shadows.

The amount of time users spent in selecting the unit of higher population density also merits discussion. In the case of units in the same class, users spent significantly more time making a decision. This could be thought of as time used wisely. Instead of guessing which of two units of identical orange color is higher, users were busy incorporating information from shadows into their decision. This effort shows in their improved performance.

The significant increase in the amount of time users took to select the unit of higher population density when comparing between different classes with illumination is a more unexpected finding. We would have predicted that users would have had faster response times, as both of the visual variables, class color and shadows resulting from changes in perspective height, are designed to lead them to the same conclusion. We speculate that users may have been using this time to integrate these visual cues.

The user study shows that the illuminated choropleth method does not interfere with map users' ability to utilize choropleth maps for the other tasks tested in the study. The shading of class colors does not prevent users from matching colors to a color legend, and does not require significantly more time. The only exceptions we found were a few places where small areas were entirely overlain and obscured by the darkest parts of these shadows. These shadows could be toned down using other illumination models that increase the diffuse brightness at the expense of the directional, but such displays would have a less noticeable shadowing effect. We would caution against using illuminated choropleth maps with clear day illumination if important areas in the map design process are identified as at risk of falling under such umbrage.

Another important functionality of choropleth maps to which our method appears to do no harm is the user's ability to compare the mean or variance of the attribute values between two regions. Our study indicates users tend to be able to estimate and compare mean values, although this takes more time with soft shadows. Our results imply that all of the additional detail provided by soft shadows is not interfering with the user's ability to synthesize a large amount of data over a region.

Our study also indicates that users tend not to be able to estimate and compare variance values. In one respect, finding similar results in selecting the region with greater variance is a good thing. It again implies our method is not decreasing the users' performance. In another respect, however, we had hoped that the information provided by soft shadows of varying lengths would improve the users' ability to compare variance among regions. We suggest that the real problem lies in the difficulty of applying the concept of variance, a decidedly more complex statistic than the mean, to a choropleth map.

# 4 Conclusions

Our illuminated choropleth method uses unclassed heights of enumeration units to cast shadows that add detail and information to classed choropleth displays. Our goal is to provide the map user with the ability to determine local relative changes between adjacent enumeration units in the same class, while not compromising the ability to compare unit colors to a legend or to compare aggregate measures between larger regions.

We note that our results test only a few tasks which a user may choose to perform with an illuminated choropleth map. We note also that the most important difference is the user's ability to differentiate between the relative population density of two *adjacent* enumeration units. Further assessment of this method could explore the ability of users to compare the relative density of *non-adjacent* polygons in a similar manner. Although much more complex, we can imagine scenarios in which such displays could prove useful. For example, if three square counties form an east-west oriented rectangle, and the western county casts a shadow on the central county, which in turn casts a shadow on the eastern county, the user could conclude that the western county is higher than the eastern. Complexities of shape and more complex variations in relative height, combined with greater numbers of polygons, would certainly complicate this task in a manner which is difficult to predict.

There are several different levels of complexity that enter into making and interpreting a choropleth map. This complexity often is a reflection of considerations that may be to some degree in potential conflict. For example, cartographers hope to reveal map patterns, but distributions of geographic data are spatially complex and may be difficult to summarize well on any map. Monmonier (1972, p. 208) points out that, even if class boundaries account for all numerical values and spatial arrangements, "…it must be recognized that the statistical and the geographical distributions are not always cooperative."

Another example of this complexity and potential conflict is reflected in the broad spectrum of studies focused on the number of choropleth classes, including ones endorsing fewer classes, more classes, and no classes. From a mapmaker's perspective, opting to create an unclassed map allows display of all potential values, as well as eliminating the assimilating duties of the mapmaker. As Muller (1980, p. 107) comments "The embedded classification implies an interpretation which is always questionable." If trying to moderate complexity, however, MacEachren's (1982, p. 31) finding that "the number of class intervals has a greater effect on complexity than does the pattern of the distribution mapped." highlights the benefit of such questionable interpretation.

A third example of this complexity and potential conflict is the way in which the map is used. A map user may not be concerned whether a map is classed or unclassed, as long as it provides useful information. Gilmarten and Shelton (1989, p. 43) point to the challenges of meeting potentially disparate needs; “Since it is not possible to predict or control whether map readers will use a choropleth map to look for regional trends or to obtain tabular data for specific units on the map (or both), the cartographer must, ideally, try to design the map so that it will fulfill both potential applications.”

We see our illuminated choropleth map as one attempt to move towards harmony. In our map, enumeration units of varying values will be represented by changes in color, shadows, or both, but this will not occur in all geographic locations. Areas of relatively high heterogeneity within classes will be highlighted by stronger shadows, and boundaries of relatively high homogeneity between classes will be demarcated by weaker shadows. Our method also incorporates visual variables from classed and unclassed maps in a manner designed to share the visual harmony of objects viewed under natural lighting. Our testing indicates users are able to effectively use this display in a local and regional sense.

We realize that our method is not without issues and may not be useful for every potential application of choropleth mapping. For example, our testing indicated that users are not able to match unit colors inside strong shadows to a legend. Also, we tested the user’s ability to pick the higher of two adjacent polygons, but not two polygons in the same class separated by some distance. As shadows generally are cast on adjacent polygons, we would assume this task to be more challenging. Also, users were tasked with identifying relative changes in population density; we would not expect our method to assist in more closely identifying absolute density values within a class.

We also realize that this illumination approach is not the only approach that could exploit the prism model to enhance choropleth maps. For example, a perspective view with hidden surfaces revealed via semi-transparency could also help to visualize the same information. An area obscured from sight by one prism might be more visible using partial transparency than another area obscured by two. We suggest this technique would face different but similarly complex challenges when compared to our method. For example, as our method allows users to identify colors and other shadows beneath soft shadows, a perspective view method may require users to identify color and changes in height on overlapping semi-transparent surfaces. Alternatively, a dynamic prism map would allow the user to interact with the display, looking at surfaces from multiple perspectives. It should be noted, however, that realistic shading and shadowing are often used to enhance such computer graphics displays. We opt to focus our methods on creating a static, planimetrically correct map.

Tobler's (1973, p. 264) concluding paragraph on unclassed choropleth maps included the following question: "If the assertion [that class intervals increase the readability of a map] is in fact valid, why then is grouping of greys into classes not also…used to enhance aerial photographs, or television?" We suggest that it is because gray shading and shadowing can add high spatial frequency information that visually complements more extensive areas of color representing information of lower spatial frequency in a visually intuitive manner.

Jenks and Caspell (1971) conclude that their research is focused on the definition, measurement, and reduction of error for classed choropleth data, but that "We have not, on the other hand, provided the cartographer with a measure of the carrying capacity of a map." Although they do not define carrying capacity or how it can be measured, they refer to "visual static" associated with an increase in the number of classes. We consider our method an attempt to limit this visual static, while at the same time enhancing the visual signal.

# 5 References

Adelson, E.H. 2000. Lightness perception and lightness illusions. In *The new cognitive neurosciences*, 2nd ed., ed. M. Gazzaniga, 339-351 Cambridge, MA: MIT Press.

___. 1993 Perceptual organization and the judgment of brightness. *Science* 262: 2042–2044.

Armstrong, M. P., N. Xiao, and D. A. Bennett. 2003. Using Genetic Algorithms to Create Multicriteria Class Intervals for Choropleth Maps. *Annals of the Association of American Geographers* 93 (3): 595-623.

Berry, B. J. L., 1968. Approaches to Regional Analysis: A Synthesis. In *Spatial Analysis: A Reader in Statistical Geography*, ed. B.L. J. Berry and D. F. Marble 24-34. Englewood Cliffs, New Jersey: Prentice-Hall.

Brassel, K.E., J. Little and T.K. Peucker. 1974. Automated Relief Representation. *Annals, Association of American Geographers* 64 (4): 610-611 and separate folded map (Map Supplement No. 17).

Brewer, C.A., 2005. *Designing better maps: A guide for GIS users*. Redlands, CA: ESRI Press, 203 pp.

___. 1994. Color use guidelines for mapping and visualization. In *Visualization in Modern Cartography*, ed. AM MacEachren and D.R.F. Taylor, 123-147. Oxford, England: Pergamon.

Brewer, C.A., G.W. Hatchard and M.A. Harrower. 2003. *ColorBrewer in print A catalog of color schemes for maps*, Cartography and Geographic Information Science 30(1), pp. 5–32.

Brewer, C.A., A.M. MacEachren, L.W. Pickle and D. Herrmann. 1997. Mapping mortality evaluating color schemes for choropleth maps. *Annals of the Association of American Geographers* 87(3)**,** 411–438.

Brewer, C.A. and L.W. Pickle. 2002. Evaluation of methods for classifying epidemiological data on choropleth maps in series. *Annals of the Association of American Geographers* 92, 662–681.

Carstensen, L.W. 1986a. Bivariate choropleth mapping: The effects of axis scaling. *The American Cartographer* 13(1): 27-42.

___. 1986b. Hypothesis testing using univariate and bivariate choropleth mapping: *The American Cartographer* 13(3): 231-251.

___. 1982. A continuous shading scheme for two variable mapping. *Cartographica* 19(3/4): 53-70.

___. 1978. Visual aspects of class intervals in choroplethic mapping. *The Cartographic Journal* 15 (1): 42–48.

Commission Internationale de l'Eclairage. 2001. Spatial distribution of daylight - CIE standard general sky. *Report CIE DS 011.1/E-2001*.

Cromley, R. G. 2005. An alternative to maximum contrast symbolization for classed choropleth maps. *Cartographic Journal* 42 (2): 137-44.

Cromley, R. G. 1996. A comparison of optimal classification strategies for choroplethic displays of spatially aggregated data. *International Journal of Geographical Information Systems* 10 (4): 404–24.

Dent, B. D. 1999. *Cartography: Thematic map design.* 5th ed. Boston: McGraw-Hill.

Dobson, M.W. 1973. Choropleth maps without class intervals? A comment. *Geographical Analysis* 5 (4): 358–60.

___. 1980. Commentary: Perception of continuously shaded maps. *Annals of the Association of American Geographers* 70 (1): 106–7.

Darula, S. and R. Kittler. 2002. CIE General sky standard defining luminance distributions. *Proceedings of IBPSA Canada eSim*.

Eyton, R. J. 1984. Map Supplement: Complementary-color, two-variable maps. *Annals of the Association of American Geographers* 74( 3) 477-490.

Fisher, W. D. 1958. On grouping for maximum homogeneity. *American Statistical Association Journal* 53:789–98.

Franklin, R., and H. R. Lewis. 1978. 3-D graphic display of discrete spatial data by prism maps. *ACM Siggraph Compter Graphics* 12 (3) 70-75.

Gilmartin, P., and E. Shelton. 1989. Choropleth maps on highresolution CRTs: The effects of number of classes and hue on communication. *Cartographica* 26 (2): 40–52.

Harrower, M.A. and C.A. Brewer. 2003. ColorBrewer.org an online tool for selecting colour schemes for maps. *British Cartographic Society* 40, pp. 27–37.

Hilbert, R., 1981. Construction and display of three-dimensional polygonal-histograms. *ACM Siggraph Compter Graphics* 15 (2) 230-241.

Horn, B. K. P. 1982. Hill shading and the reflectance map. *Geo-Processing* 2: 65-146.

Imhof, E. 1982. *Cartographic relief presentation*. Walter de Gruyter, Berlin and New York.

Jenks, G. F. 1977. Optimal data classification for choropleth maps. *Occasional Paper no. 2*. Lawrence, Kansas: Department of Geography, University of Kansas.

Jenks, G. F., and D. A. Brown. 1966. Three-Dimensional Map Construction. *Science* 154 (3751): 857-864.

Jenks, G. F., and F. C. Caspall. 1971. Error on choroplethic maps: Definition, measurement, reduction. *Annals of the Association of American Geographers* 61 (2): 217–44.

Jensen, J. 1978. Three-Dimensional Choropleth Maps: Development and Aspects of Cartographic Communication. *The Canadian Cartographer*, 15(2): 123-141.

Kennelly, P., and A. J. Kimerling 2004. Hillshading of Terrain Using Layer Tints with Aspect-Variant Luminosity. *Cartography and Geographic Information Science*, 31(2), 67-77.

Kennelly, P., and J. Stewart 2006. A Uniform Sky Model to Enhance Shading of Terrain and Urban Elevation Models. *Cartography and Geographic Information Science,* 33(1), 21-36.

Lavin, S. and J.C. Archer. 1984. Computer-produced unclassed bivariate choropleth maps. *The American Cartographer* 11(1): 49-57.

MacEachren, A. M. 1982. Map complexity: Comparison and measurement. *The American Cartographer* 9 (1): 31–46.

MacEachren, A.M., C.A. Brewer and L.W. Pickle. 1998. Visualizing georeferenced data representing reliability of health statistics, *Environment and Planning A* 30, 1547–1561.

Mak, K., and M. R. C. Coulson. 1991. Map user response to computer-generated choropleth maps: Comparative experiments in classification and symbolization. *Cartography and Geographic Information Systems* 18 (2): 109–24.

Mersey, J. E. 1990. Colour and thematic map design: The role of colour scheme and map complexity in choropleth map communication. *Cartographica Monograph no. 41*. Toronto: University of Toronto Press.

Monmonier, M. S. 1972. Contiguity-biased class-interval selection: A method for simplifying patterns on statistical maps. *Geographical Review* 62:203–28.

Moon, P. and D. Spencer. 1942, Illumination from a non-uniform sky. *Illuminating Engineering* 37: 707-726.

Muller, J.-C. 1979. Perception of continuously shaded maps. *Annals of the Association of American Geographers* 69 (2): 240–49.

Muller, J.-C. 1980. Comment in reply. *Annals of the Association of American Geographers* 70 (1): 107-08.

Olson, J.M. 1981. Spectrally encoded two-variable maps. *Annals of the Association of American Geographers* 71 (2): 259-276.

___. 1975. Autocorrelation and visual map complexity. *Annals of the Association of American Geographers* 65 (2): 189–204.

Olson, J.M. and C.A. Brewer. 1997. An evaluation of color selections to accommodate map users with color-vision impairments, *Annals of the Association of American Geographers* 87, 103–134.

Peucker, T.K., M. Tichenor, and W.D. Rase. 1974. The Computer Version of Three Relief Representations. In J.C. Davis and M. McCullagh, (eds.) *Display and Analysis of Spatial Data*, New York: John Wiley & Sons, (187-197).

Robinson, A. H. 1982. *Early thematic mapping in the history of cartography*. Chicago: University of Chicago.

Robinson, A. H., J. L. Morrison, P. C. Muehrcke, A. J. Kimerling, and S. C. Guptill. 1995. *Elements of cartography*. 6th ed. New York: JohnWiley and Sons.

Slocum, T., McMaster, R., Kessler, F., Howard H., 2008. *Thematic cartography and geographic visualization*, 3rd ed. Pearson Prentice–Hall, Inc., Upper Saddle River, New Jersey.

Tobler, W. 1973. Choropleth maps without class intervals. *Geographical Analysis* 5 (3): 262–65.

Xiao, N. and M. P. Armstrong. 2005. Supporting the Comparison of Choropleth Maps Using an Evolutionary Algorithm. *Cartography and Geographic Information Science* 32 (4): 347-358.

Yoeli, P. 1965. Analytical Hill Shading. *Surveying and Mapping* 25(4):573- 579.

Yoeli, P. 1966. Analytical Hill Shading and Density. *Surveying and Mapping* 26(2):253-259.

Yoeli, P. 1967. The Mechanisation of Analytical Hill Shading. *Cartographic Journal* 4:82-88.

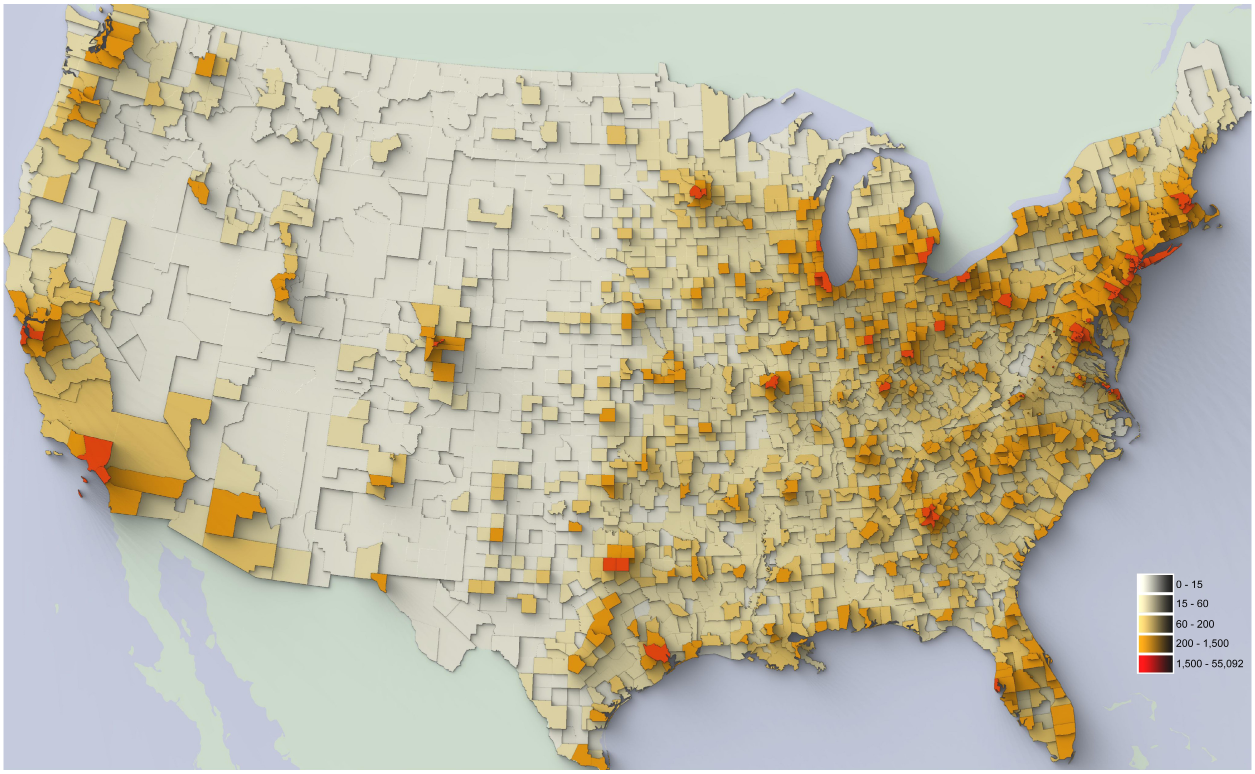



**Figure 1**: An illuminated choropleth map showing population density of counties in the conterminous United States.

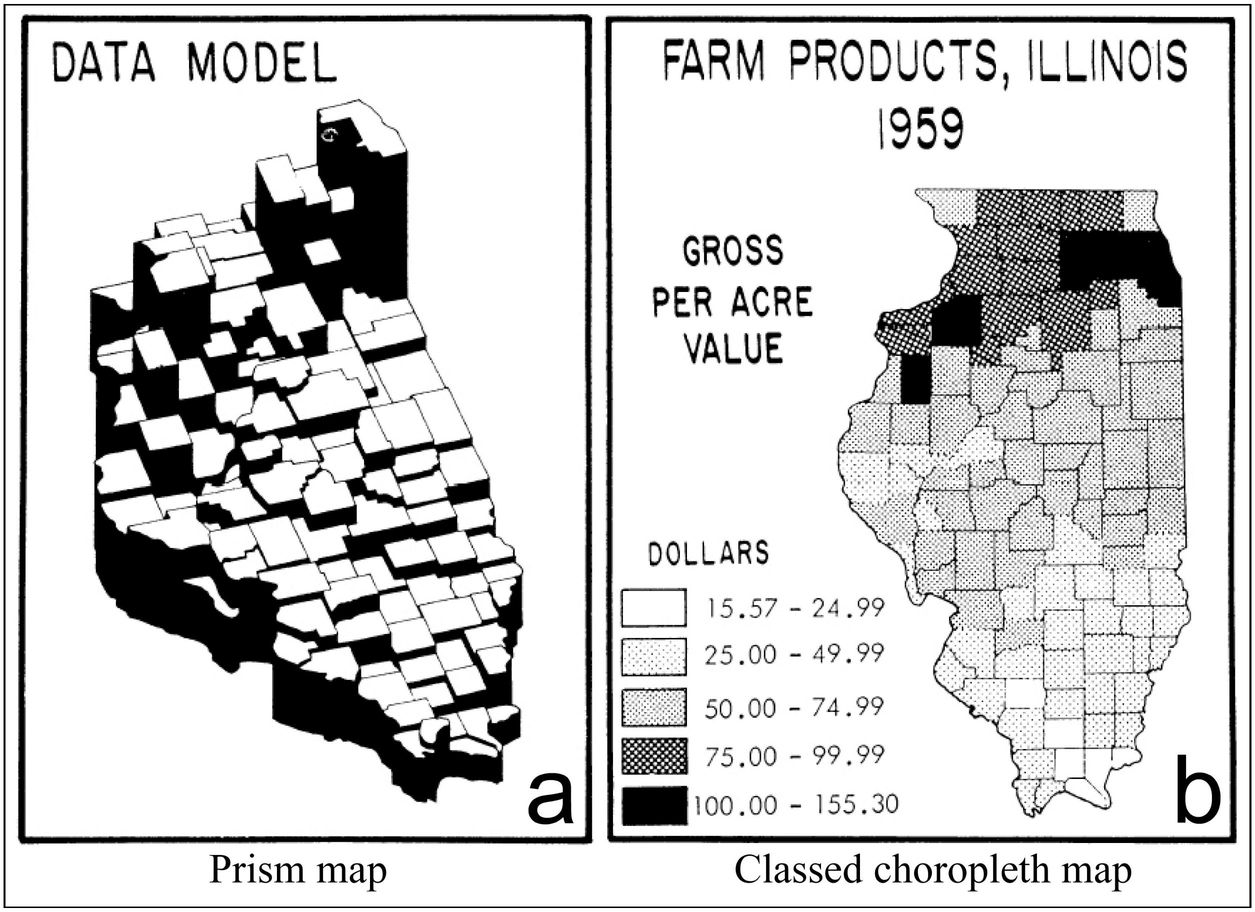



**Figure 2**: A comparison of a prism map with a classed choropleth map (from Jenks and Caspall, 1971. Used with permission from the Association of American Geographers).

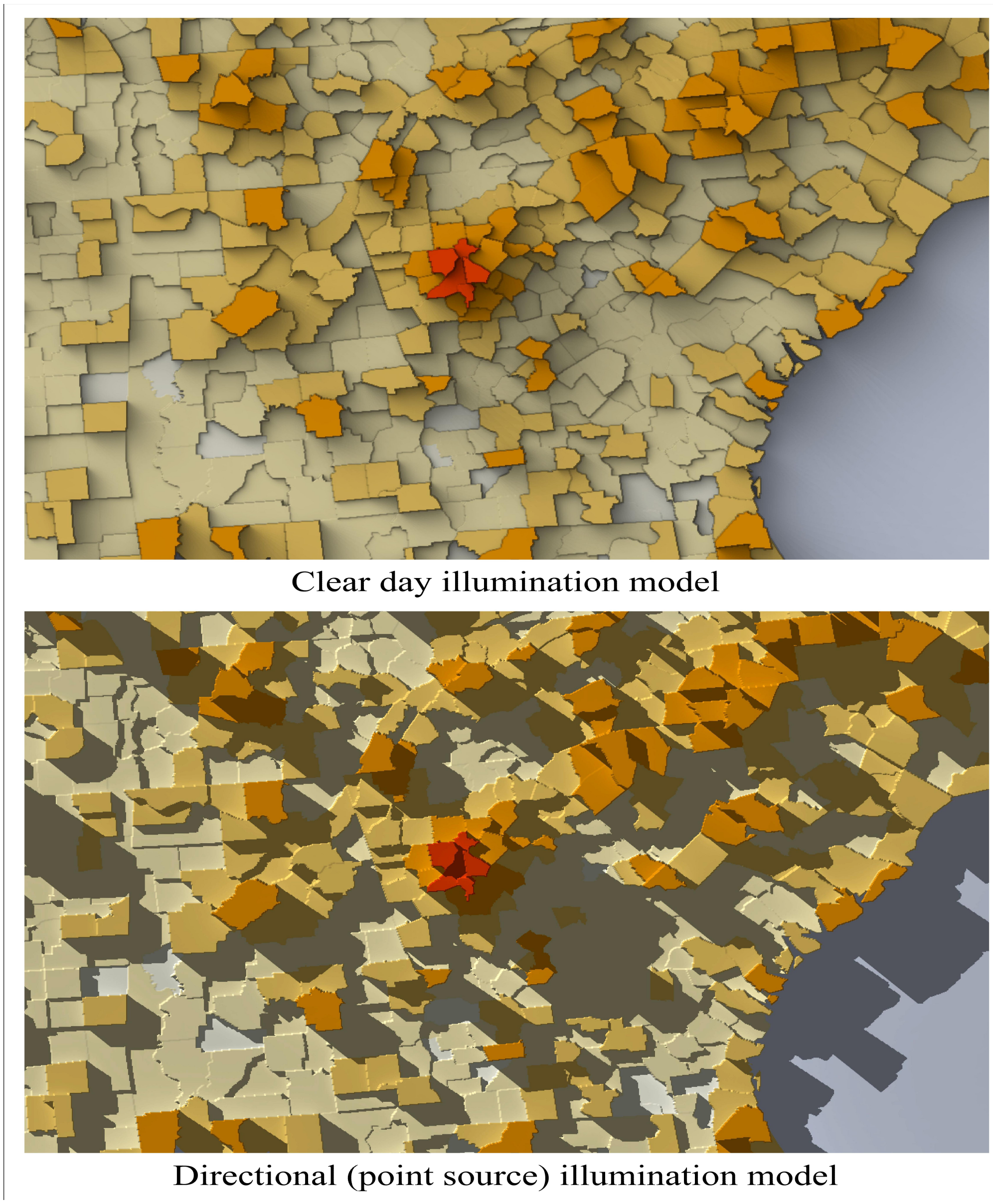



**Figure 3**: A comparison of illuminated choropleth maps with a clear day illumination model and a directional (point source) illumination model.

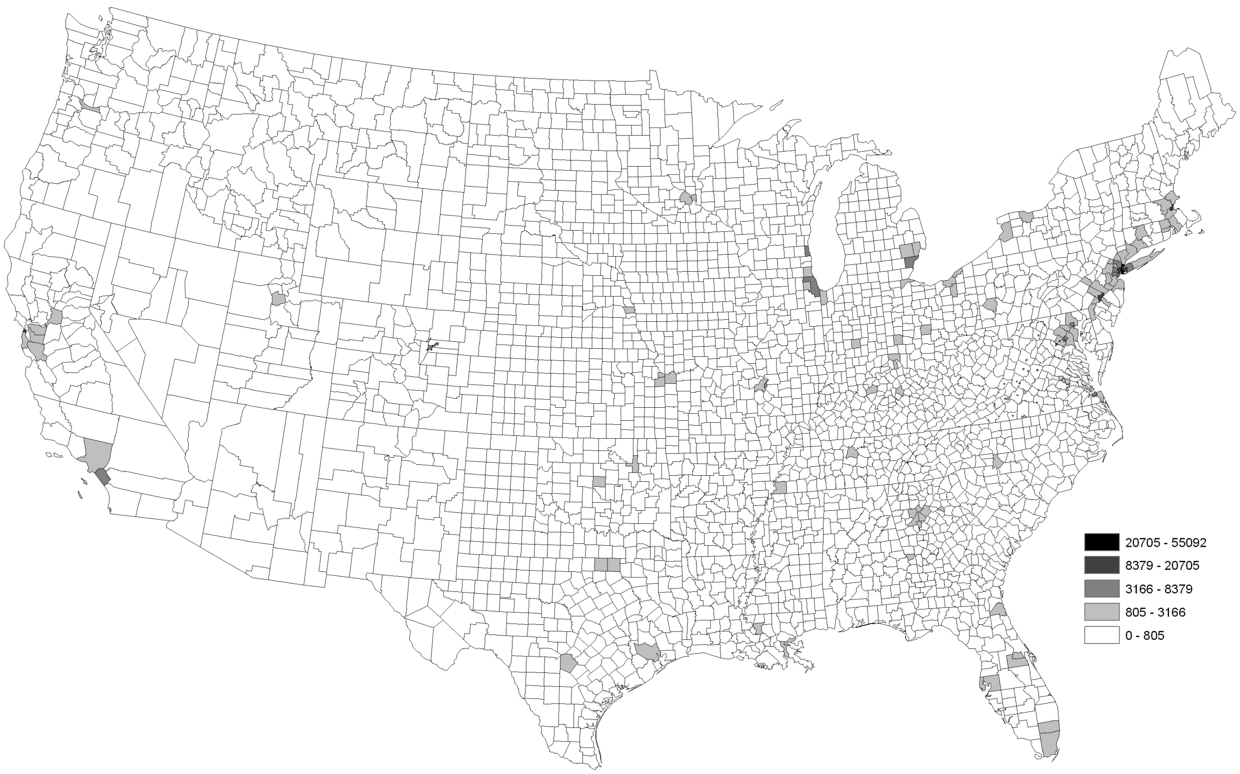



**Figure 4**: A classed population density map of counties in the conterminous United States using Jenks' optimal method.

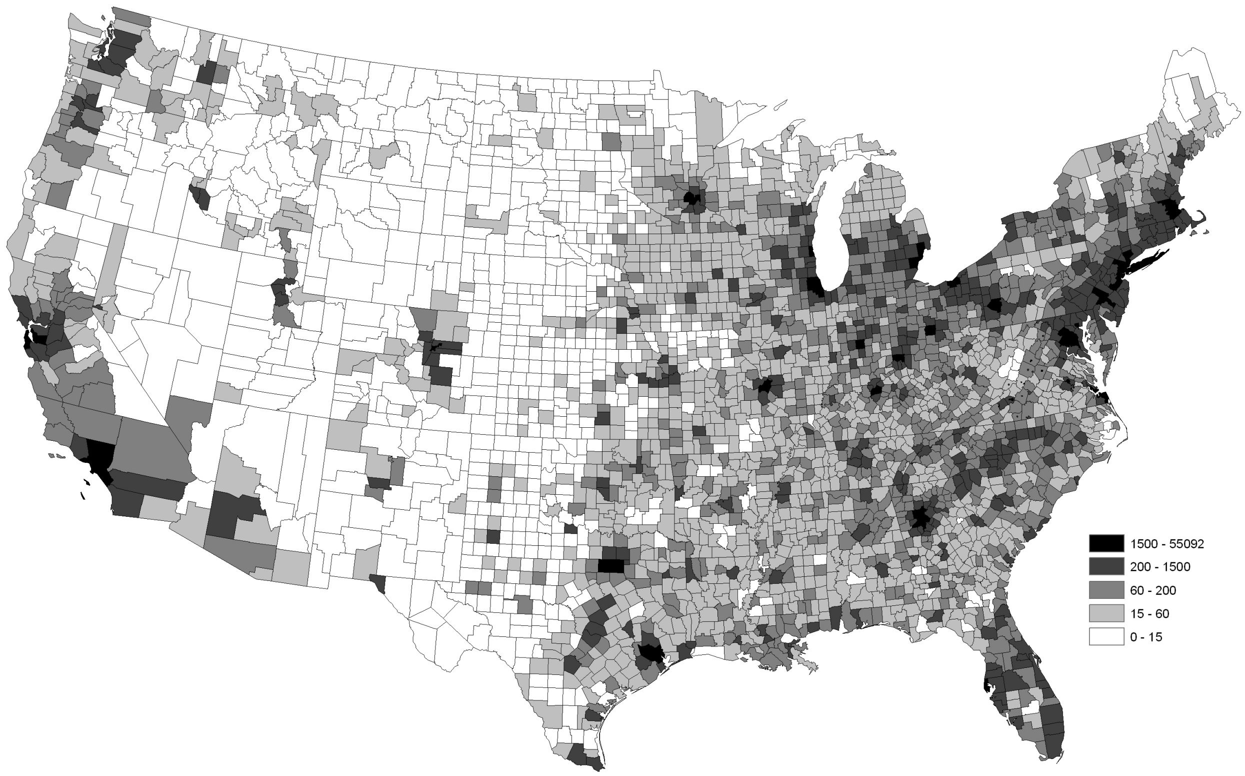



**Figure 5**: A classed population density map of counties in the conterminous United States using class breaks that provide a wider distribution of more densely populated classes.

**Figure 6**: An unclassed map showing a prism model of county population density, with heights normalized by an exponential function in **Figure 7**.  The  prism model is shaded according to a clear day illumination model.

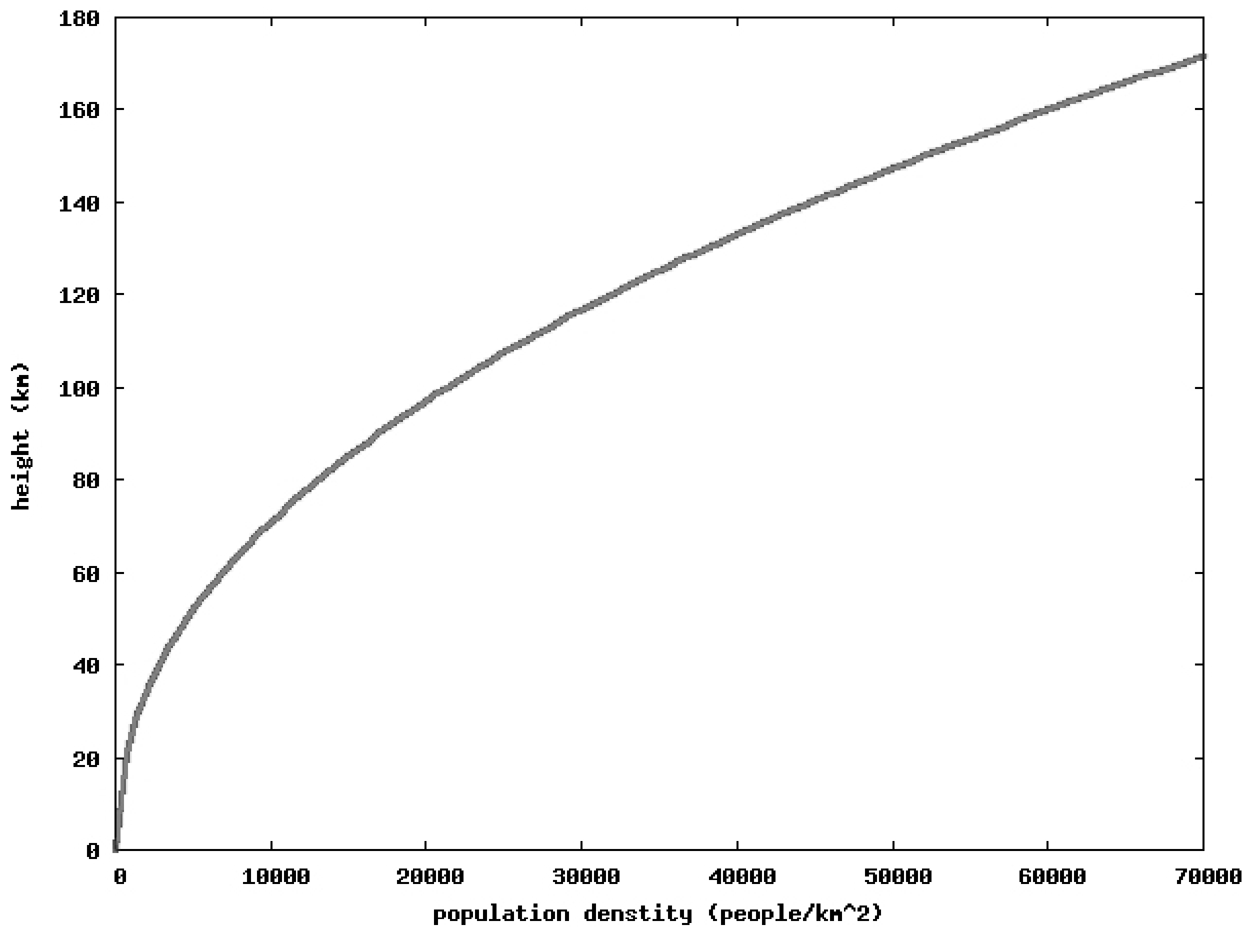
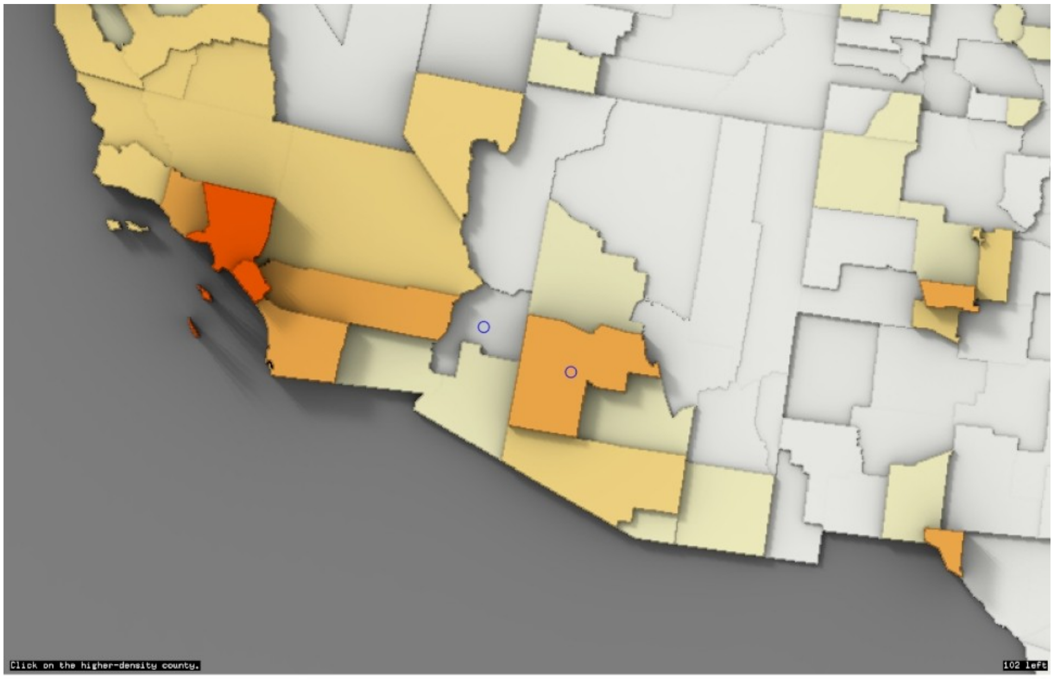
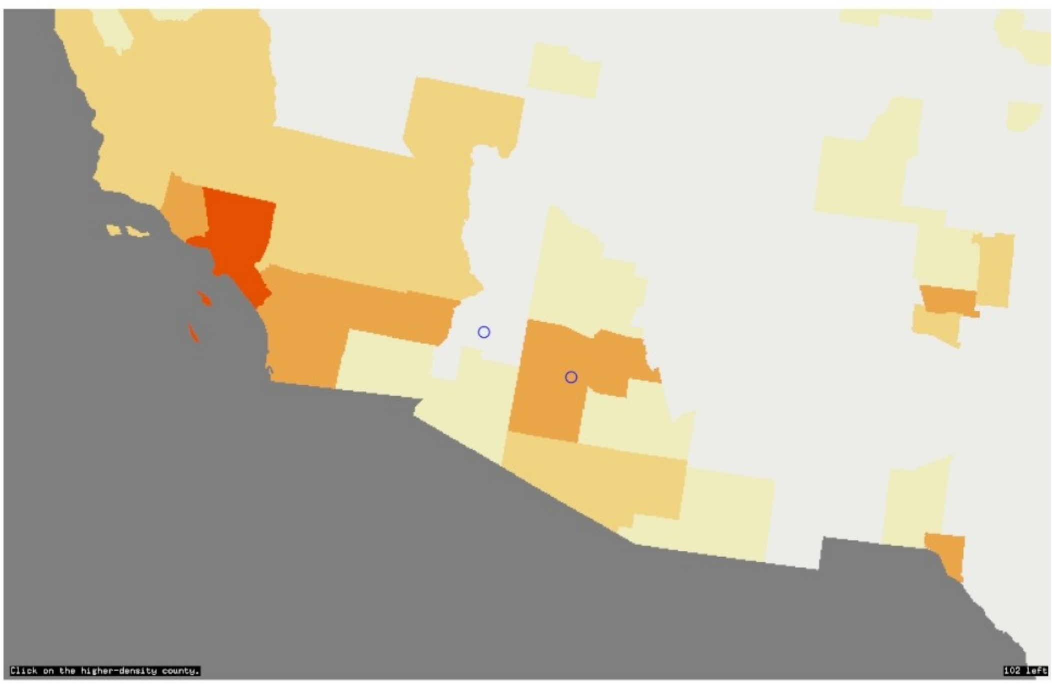



**Figure 7**: A graph of prism height vs. population density, illustrating the exponential function used to create a more even distribution of population density values for illumination modeling.

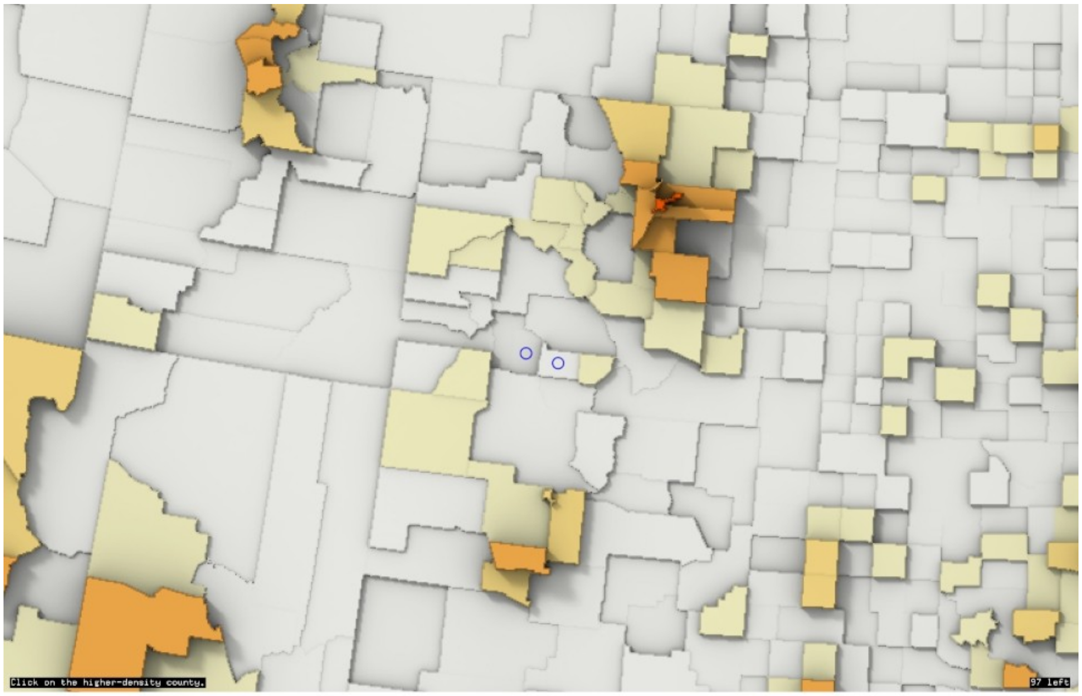



selecting between counties in *different classes, with illumination*

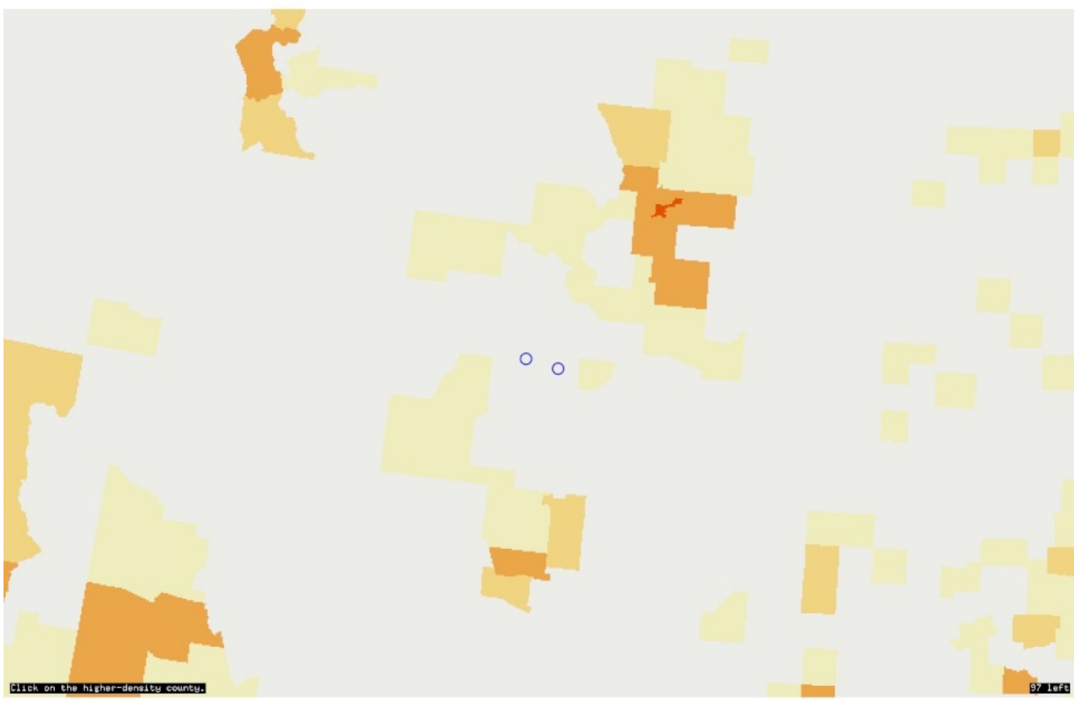



selecting between counties in *different classes, without illumination*

**Figure 8**: Selection Mode (Different Classes): The subject was asked to click on the denser of the two counties in different classes indicated with the blue circles near the center of the screen.

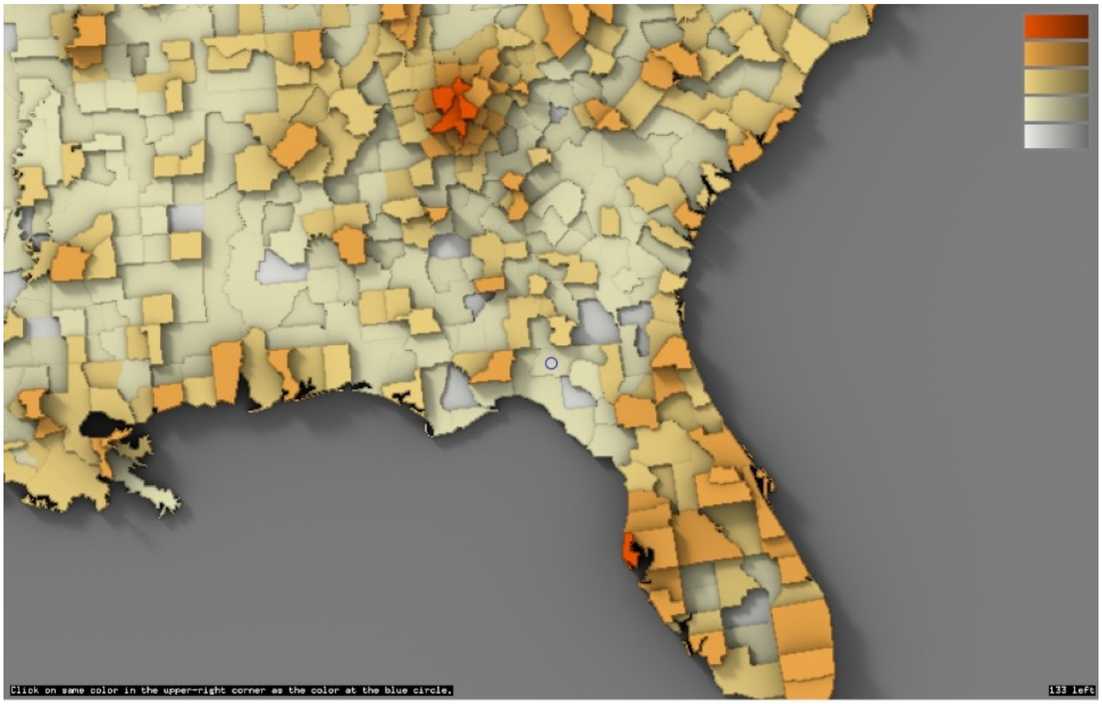


selecting between counties in *the same class, without illumination*

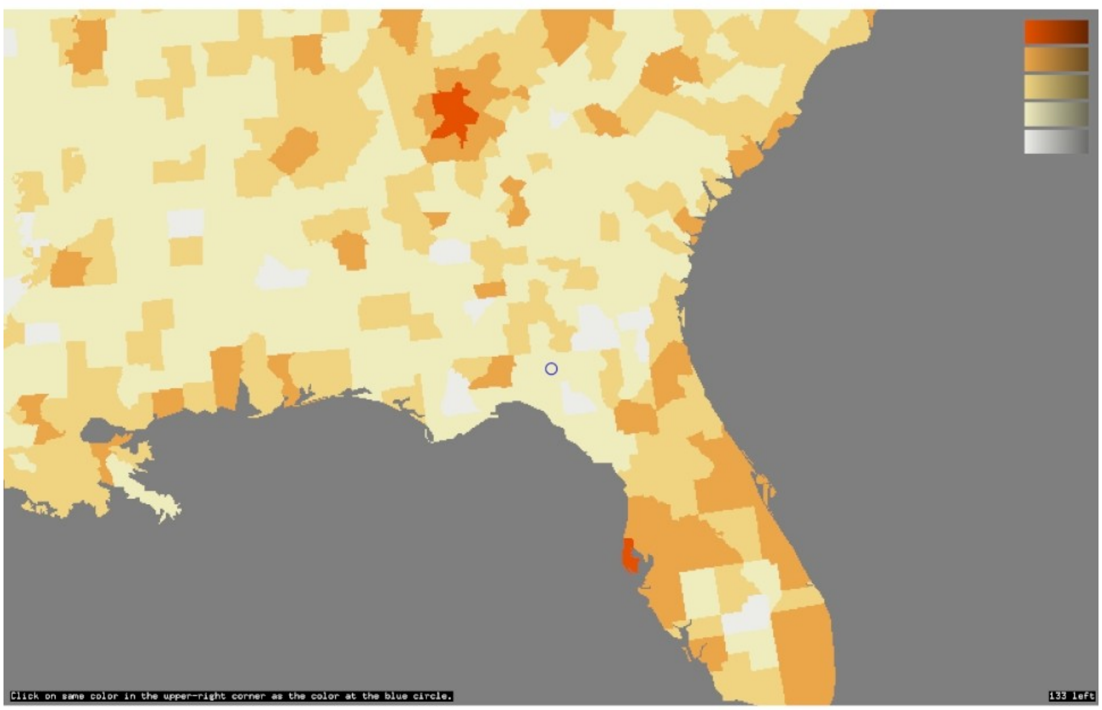


selecting between counties in *the same class, without illumination*

**Figure 9**: Selection Mode (Same Class): The subject was asked to click on the denser of the two counties in the same class indicated with the blue circles near the center of the screen.

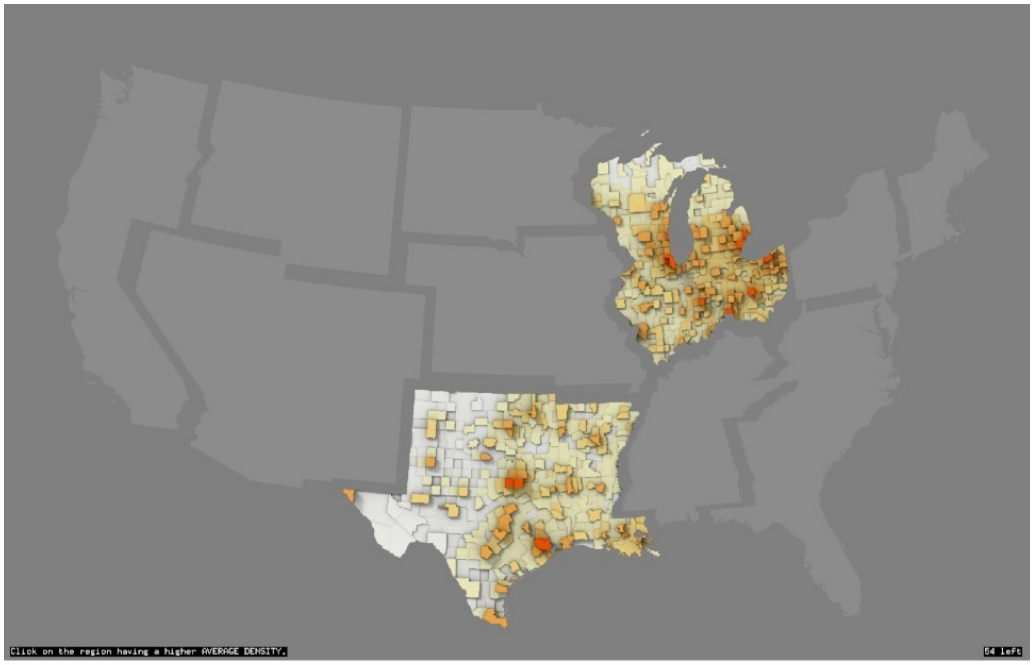



Task with illumination

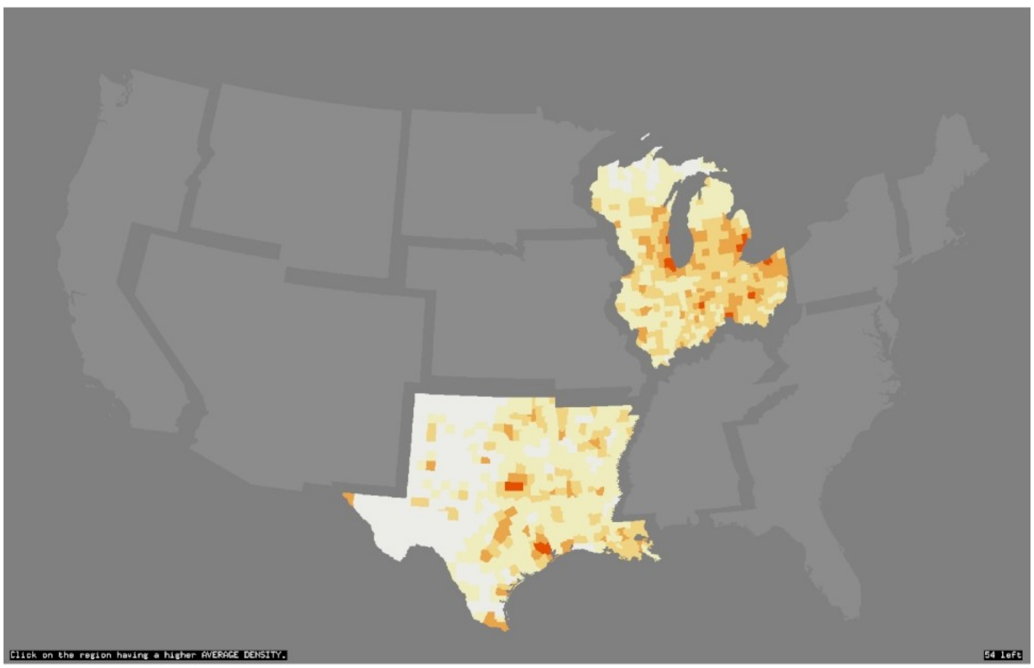



Task without illumination

**Figure 10**: Legend Matching Task: The subject was asked to click on the legend color that matched that of the county with the blue circle near the center of the screen.




Task with illumination




Task without illumination

**Figure 11**: Region Mean and Variance Modes: The subject was asked to click on the region that had greater mean population density (in one test) or greater variance in population density (in another test).

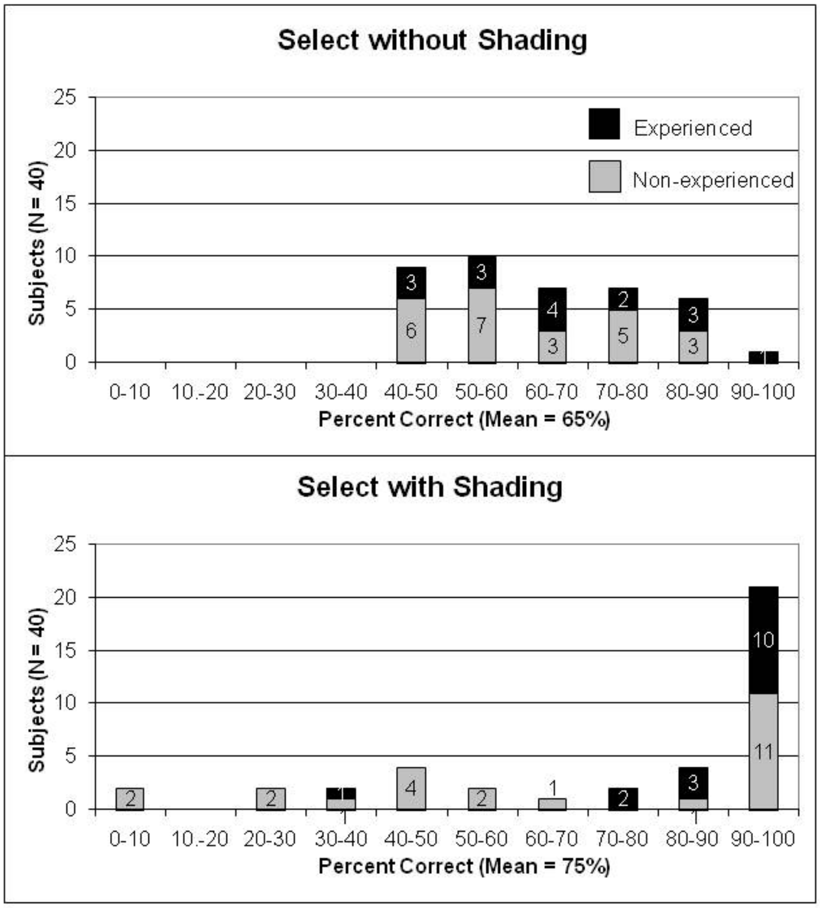



**Figure 12**: Histograms showing performance of subjects in selecting more densely populated of adjacent counties in the same class, without and with shading. Black sections of the bars represent users experienced with shading; gray sections represent those without experience.

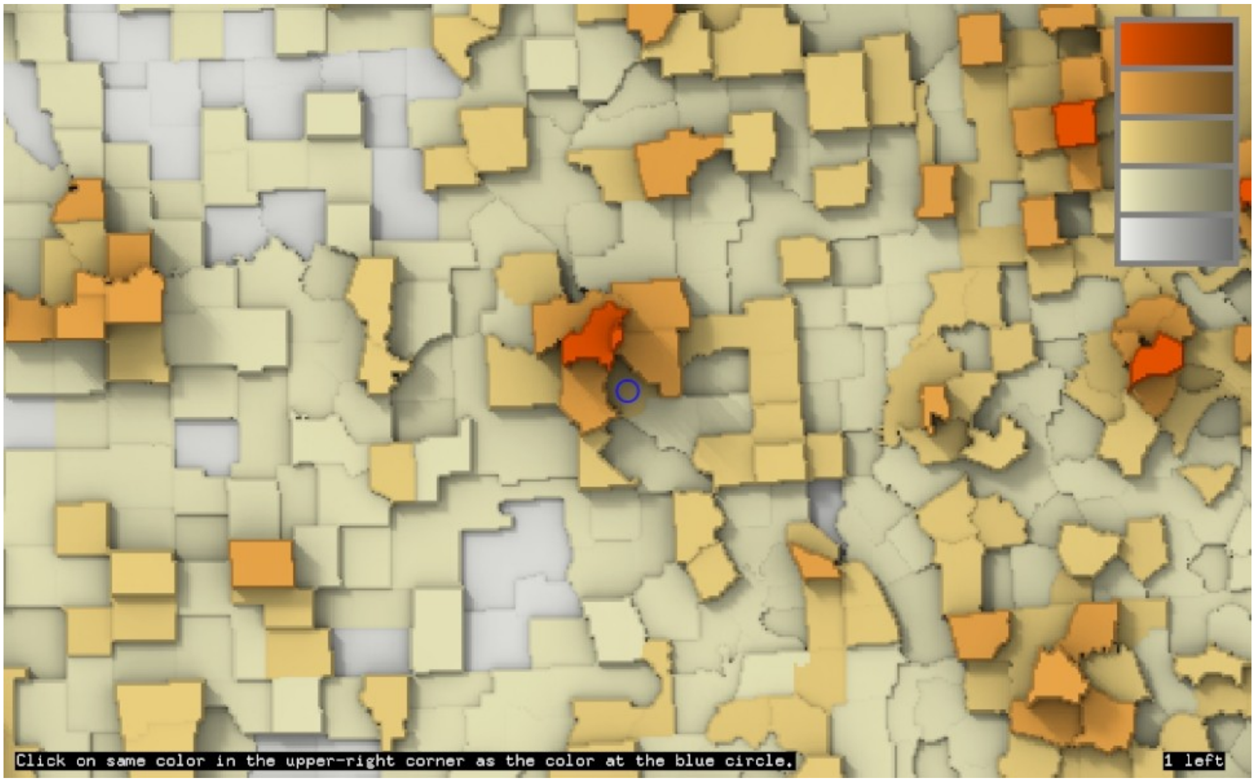



**Figure 13**:  An example in which matching is difficult because the county being classified (indicated above with the blue circles) is southeast of a county with much greater population density, which casts a shadow upon it.

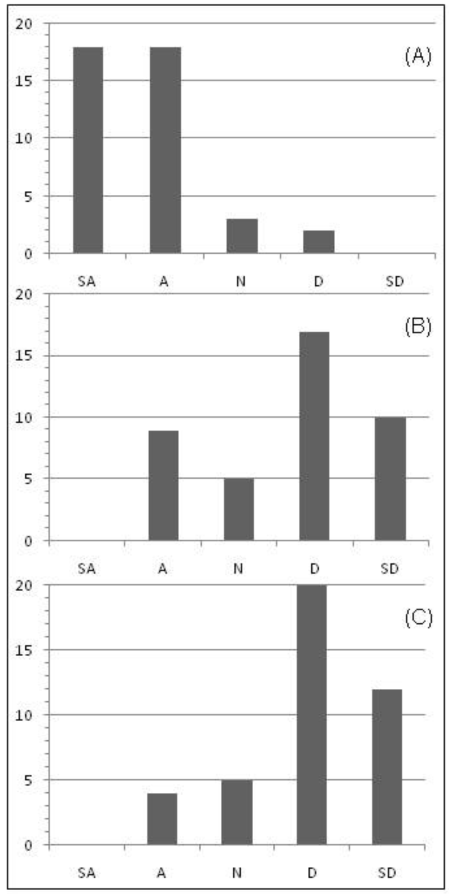



**Figure 14**: Participant responses to the post-trial survey. See text for statements (A), (B), and (C).